EXERCISE – V HINTS & SOLUTIONS

Sol.1 **A**

$x = t^2 + t + 1$

$y = t^2 - t + 1$; $\frac{x-y}{2} = t$; $\frac{x+y}{2} = t^2 + 1$

Eliminate t

$2(x + y) = (x - y)^2 + 4$

It is a perfect square. So it is a parabola (A)

(b) $\phi = \frac{\pi}{2} - \theta$

Normal at θ, ϕ passes through (h, k)

$\therefore$ ah $\cos\theta$ + bk $\cot\theta = a^2 + b^2$

ah $\sin\theta$ + bk $\tan\theta = a^2 + b^2$

Eliminate h

$bk(\cot\theta\sin\theta - \tan\theta\cos\theta) = (a^2 + b^2)(\sin\theta - \cos\theta)$

so $k = -\frac{(a^2+b^2)}{b}$

(c) c.o.c. with respect to (h, k)

$hx - ky = 9$

compaire with x = 9

h = 1, k = 0

(1, 0)

pair of Tangents

$SS_1 = T^2$

$(x^2 - y^2 - 9)(1 + 0 - 9) = (x - 9)^2$

$-8x^2 + 8y^2 + 72 = x^2 - 18x + 81$

$9x^2 - 8y^2 - 18x + 9 = 0$

Sol.2 Tangent to the parabola $y^2 = 8x$

$y = mx + \frac{2}{m}$ ...(1)

It will also touch xy = –1

$x\left(mx + \frac{2}{m}\right) = -1$

Has equal roots

$\frac{4}{m^2} = 4m \Rightarrow m = 1$

So y = x + 2

Sol.3 $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$ for $\alpha \in \left(0, \frac{\pi}{2}\right)$

$\sin^2\alpha = \cos^2\alpha(e^2 - 1)$

$\cos^2\alpha\, e^2 = 1$

absicissa of foci ($\cos\alpha$ e, 0) does not change.

Sol.4 $2x + \sqrt{6}y = 2$ ...(1)

Hyp. $x^2 - 2y^2 = 4$

Equation of tangent

$xx_1 - 2yy_1 = 4$

Comparing with (1)

$x_1 = 4$ and $y_1 = -\sqrt{6}$

Sol.5 Any point on the Hyp. $\frac{x^2}{8} - \frac{y^2}{4} = 1$ is

($3\sec\theta$, $2\tan\theta$) c.o.c. of the circle $x^2 + y^2 = 9$

w.r.t. ($3\sec\theta$, $2\tan\theta$) is

$3\sec\theta \cdot x + 2\tan\theta\, y = 9$ ...(1)

Let (h, k) be the mid point M (h, k)

Equation of chord with given mid point

$T = S_1$

$xh + ky = h^2 + k^2$ ...(2)

Since (1) and (2) represent the same line

$\frac{3\sec\theta}{h} = \frac{2\tan\theta}{k} = \frac{9}{h^2+k^2}$

$\sec\theta = \frac{9h}{3(h^2+k^2)}$; $\tan\theta = \frac{9k}{2(h^2+k^2)}$

Hence $\frac{81h^2}{9(h^2+k^2)} - \frac{81k^2}{4(h^2+k^2)} = 1$

Reqd. locus $\frac{x^2}{9} - \frac{y^2}{4} = \left(\frac{x^2+y^2}{9}\right)^2$

Sol.6 $\frac{x^2}{25} + \frac{y^2}{16} = 1$

Ecentricity = $\frac{3}{5}$

Ecentricity = $\frac{5}{3}$ and it passes through (± 3, 0)

$\Rightarrow$ its equation $\frac{x^2}{9} - \frac{y^2}{b^2} = 1$

where $1 + \frac{b^2}{9} = \frac{25}{9} \Rightarrow b^2 = 16$

$\Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1$ and its foci are (± 5, 0)

Sol.7 **(a)** Let A, B, C and D be the complex Number $\sqrt{2}$, $-\sqrt{2}$, $\sqrt{2}$ i, $-\sqrt{2}$ i respectively.,

$$\Rightarrow \frac{PA^2 + PB^2 + PC^2 + PD^2}{QA^2 + QB^2 + QC^2 + QD^2}$$

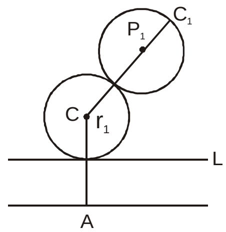
$$= \frac{|z_1 - \sqrt{2}|^2 + |z_1 + \sqrt{2}|^2 + |z_1 + \sqrt{2}i|^2 + |z_1 - \sqrt{2}i|^2}{|z_2 + \sqrt{2}|^2 + |z_2 - \sqrt{2}|^2 + |z_2 - \sqrt{2}i| + |z_2 + \sqrt{2}i|^2}$$

$$= \frac{|z_1|^2 + 2}{|z_2|^2 + 2} = \frac{3}{4}$$

(b) Let c be the centre of the required circle. Draw a line parallel to L at a distance of r_1 from it from it

Now $CP_1 = AC$

$\Rightarrow$ C lies oin a parabola



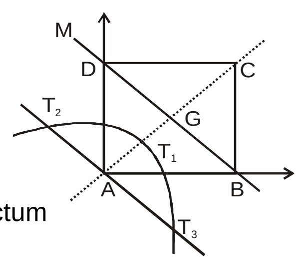
(c) Locus of S will be parabola.

$\because$ AG = $\sqrt{2}$

$$AT_1 = T_1G = \frac{1}{\sqrt{2}}$$

As A is the focus, T_1 is the vertex and BD is the direction of parabola]

Also T_2T_3 is latus rectum



$$\therefore\ T_2T_3 = 4 \times \frac{1}{\sqrt{2}}$$

$$\text{Area of } \Delta T_1T_2T_3 = \frac{1}{2} \times \frac{1}{\sqrt{2}} \times \frac{4}{\sqrt{2}} = 1$$

Sol.8 **(a)** Given $\frac{x^2}{4} + \frac{y^2}{3} = 1$

$\Rightarrow$ a = 2 , b = $\sqrt{3}$ $\Rightarrow$ e = $\frac{1}{2}$

So that ae = 1

hence the ecentricity e_1 of hyp. is given by

$1 = e_1 \sin\theta \Rightarrow e_1 = \text{cosec}\theta$

$$\Rightarrow b^2 = \sin^2\theta \left(\frac{1 - \sin^2\theta}{\sin^2\theta}\right) = \cos^2\theta$$

Hence the hyp.

$$\frac{x^2}{\sin^2\theta} - \frac{y^2}{\cos^2\theta} = 1$$

$\Rightarrow x^2 \text{cosec}^2\theta - y^2 \sec^2\theta = 1$

(b) (A) When two circle are intersecting they have a common normal and tangent (P)(Q).

(B) Two mutually enternal circles have a common normal and tangent (P)(Q).

(C) When one circle lies inside of other then, they have a common normal but no common tangent. (Q)(R)

(D) Two branches of a hyp. have a common normal but no common tangent. (Q)(R).

Sol.9 **(a)** $(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$

$ax^2 + by^2 + c = 0$ or $x^2 - 5xy + 6y^2 = 0$

$x^2 + y^2 = \left(\frac{-c}{a}\right)$; $x - 2y = 0$ and $x - 3y = 0$

If a = b

Hence the given equations represent two straight lines and a circle, when a = b and c is of sign opposite to that of a

(b) Hyp. is $\frac{(x-\sqrt{2})^2}{4} - \frac{(y+\sqrt{2})^2}{2} = 1$

a = 2, b = $\sqrt{2}$

$e = \sqrt{\frac{3}{2}}$

$$\text{Area} = \frac{1}{2} a(e-1) \times \frac{b^2}{a}$$

$$= \frac{1}{2} 2 \left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}}\right) \times \frac{2}{2}$$

$$= \frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}} = \sqrt{\frac{3}{2}} - 1$$

Sol.10 **(P)** $\left|\frac{0+0-1}{\sqrt{h^2+k^2}}\right| = 2$

$h^2 + k^2 = \left(\frac{1}{2}\right)^2$ which is a circle

(Q) If $|z - z_1| - |z - z_2| = k$ where $k < |z_1 - z_2|$ the locus is a hyperbola.

(R) Let t = tan α

x = $\sqrt{3}$ cos 2α, y = sin 2α

$\frac{x^2}{3} + y^2 = 1$ Ellipse

(S) e>1 for a hyperbola If e = 1 then parabola

(T) Let z = x + iy

$(x + 1)^2 - y^2 = x^2 + y^2 + 1$

$y^2 = x$, parabola

Sol.11 Ellipse and hyperbola will be confocal

$\Rightarrow$ $(\pm ae, 0) = (\pm 1, 0)$

$\Rightarrow$ $\left(\pm a \times \frac{1}{\sqrt{2}}, 0\right) = (\pm 1, 0)$

$a = \sqrt{2}$ and $e = \frac{1}{\sqrt{2}}$

$b^2 = a^2 (1 - e^2) \Rightarrow b^2 = 1$

Ellipse $\frac{x^2}{2} + \frac{y^2}{1} = 1$

Sol.12 Tangent to ellipse $\frac{x^2}{9} - \frac{y^2}{4} = 1$ is

$y = mx + \sqrt{9m^2 - 4}$, $m > 0$

It is tangent to $x^2 + y^2 - 8x = 0$

$$\frac{4m + \sqrt{9m^2 - 4}}{\sqrt{1 + m^2}} = 4$$

$495 m^4 + 104m^2 - 400 = 0$

$m^2 = \frac{4}{5} \Rightarrow m = \frac{2}{\sqrt{5}}$

Tangent is $y = \frac{2}{\sqrt{5}} x + \frac{4}{\sqrt{5}}$

$\Rightarrow 2x - \sqrt{5} y + 4 = 0$

Sol.13 A point on hyperbola is $(3 \sec \theta, 2 \tan \theta)$

It is lie on circle so

$9 \sec^2 \theta + 4 \tan^2 \theta - 24 \sec \theta = 0$

$\Rightarrow$ $13 \sec^2 \theta - 24 \sec \theta - 4 = 0$

$\Rightarrow$ $\sec \theta = 2, -\frac{2}{13}$

$\therefore$ $\sec \theta = 2 \Rightarrow \tan \theta = \sqrt{3}$

POI are $A(6, 2\sqrt{3})$ and $B(6, -2\sqrt{3})$

equation of circle

$(x - 6)^2 + y^2 = (2\sqrt{3})^2 \Rightarrow x^2 + y^2 - 12x + 24 = 0$

Sol.14 Substituting $\left(\frac{a}{e}, 0\right)$ in $y = -2x + 1$

$0 = -\frac{2a}{e} + 1 \Rightarrow \frac{2a}{e} = 1 \Rightarrow a = \frac{e}{2}$

$c^2 = a^2m^2 - b^2$

$1 = 4a^2 - b^2$

$1 = \frac{4e^2}{4} - b^2$

$b^2 = e^2 - 1$

Also $b^2 = a^2 (e^2 - 1)$

$a = 1$

$\Rightarrow$ $e = 2$

Sol.15 **B,D**

$\frac{x^2}{4} + y^2 = 1$

$e = \sqrt{3}/2$

so eccenticity of hyperbola $= 2/\sqrt{3}$

$\Rightarrow 1 + \frac{b^2}{a^2} = \frac{4}{3} \Rightarrow 3b^2 = a^2$

.....(1)

the equation of H.B. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$,

passes through $(\pm\sqrt{3}, 0)$ $\Rightarrow b^2 = 1$

so equation of hyperbola $\frac{x^2}{3} - y^2 = 1$

Sol.16 **B**

Equation of normal at P(6, 3)

$y - 3 = -\frac{3}{6} \cdot \frac{a^2}{b^2} (x - 6)$

put $y = 0$, $\Rightarrow \frac{6b^2}{a^2} + 6 = 9 \Rightarrow 2b^2 = a^2$

$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{\frac{3}{2}}$